CAS Course 3 - Actuarial Models

Before commencing study for this four-hour, multiple-choice examination, candidates should read the introduction to Materials for Study. Items marked with a bold W are available at no charge under Web Notes.

Please check the Admissions section of the CAS Web Site for any changes to the *Syllabus*.

The CAS will grant credit for CAS Exam 3 to those who successfully complete SOA Course 3 in the current education structure.

This examination develops the candidate's knowledge of the theoretical basis of actuarial models and the application of those models to insurance and other financial risks. A thorough knowledge of calculus, probability, and interest theory is assumed. Knowledge of risk management at the level of Exam 1 is also assumed.

The candidate will be required to understand, in an actuarial context, what is meant by the word "model," how and why models are used, and their advantages and their limitations. The candidate will be expected to understand what important results can be obtained from these models for the purpose of making business decisions, and what approaches can be used to determine these results.

A variety of tables will be provided to the candidate with the exam. They include values for the standard normal distribution, illustrative life tables, and abridged inventories of discrete and continuous probability distributions. Since they will be included with the examination, candidates will not be allowed to bring copies of the tables into the examination room.

The CAS will test the candidate's knowledge of the material, but may decide not to include questions from every reading on a particular exam. A guessing adjustment will be used in scoring Exam 3.

A. Contingent Payment Models and Survival Models

Range of weight for Section A: 25-30 percent

Candidates should be able to work with discrete and continuous univariate probability distributions for failure time random variables. They will be expected to set up and solve equations in terms of life table functions, cumulative distribution functions, survival functions, probability density functions, and hazard functions (e.g., force of mortality), as appropriate. They should have similar facility with models of the joint distribution of two failure times (multiple lives) and the joint distribution of competing risks (multiple decrement). They should be able to formulate and apply stochastic and deterministic models for the present value of a set of future contingent cash flows under an assumed interest rate structure. Candidates also should be able to apply the equivalence principle, and other principles in the text, to associate a cost or pattern of (possibly contingent) costs with a set of future contingent cash flows.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Create stochastic and deterministic models for present value, with an assumed interest rate structure, of a set of future contingent cash flows. *Range of weight: 3-7 percent*	b. Deterministic interest rate structure c. Scheme for the amounts of the cash flows d. Probability distribution of the times of the cash flows e. Probability distribution of the present value of the set of cash flows

2. Calculate the effects of changes to the components of the model. *Range of weight: 0-5 percent*	b. Deterministic interest rate structure c. Scheme for the amounts of the cash flows d. Probability distribution of the times of the cash flows e. Probability distribution of the present value of the set of cash flows
3. Apply a principle to a present value model to associate a cost or pattern of costs (possibly contingent) with a set of future contingent cash flows. *Range of weight: 3-7 percent*	c. Principles include: equivalence, exponential, standard deviation, variance, and percentile d. Models include: present value models based on 4-6 below e. Applications include: insurance, health care, credit risk, environmental risk, consumer behavior (e.g., subscriptions), and warranties
4. For discrete and continuous univariate probability distributions for failure time random variables, develop expressions in terms of the life table functions, l_x, q_x, p_x, ${}_nq_x$, ${}_np_x$, and ${}_{m\|n}q_x$, for the cumulative distribution function, the survival function, the probability density function and the hazard function (force of mortality), and be able to: • Establish relations between the different functions • Develop expressions, including recursion relations, in terms of the functions for probabilities and moments associated with functions of failure time random variables, and calculate such quantities using simple failure time distributions • Express the impact of explanatory variables on a failure time distribution in terms of proportional hazards and	d. Failure time random variables e. Life table functions f. Cumulative distribution functions g. Survival functions h. Probability density functions i. Hazard functions j. Relationships between the above variables in the above functions

accelerated failure time models *Range of weight: 3-7 percent*	
5. Given the joint distribution of two failure times, be able to: • Calculate probabilities and moments associated with functions of these random variables • Characterize the distribution of the smaller failure time (the joint life status) and the larger failure time (the last survivor status) in terms of functions analogous to those in Learning Objective A4, as appropriate • Develop expressions, including recursion relations, for probabilities and moments of functions of the joint life status and the last survivor status, and express these in terms of the univariate functions in Learning Objective A4 in the case in which the two failure times are independent *Range of weight: 3-7 percent*	e. Joint distribution of failure times f. Probabilities and moments
6. Based on the joint distribution (pdf and cdf) of the time until failure and the cause of failure in the competing risk (multiple decrement) model, in terms of the functions $l_x^{(t)}$, ${}_tq_x^{(t)}$, ${}_tp_x^{(t)}$, ${}_td_x^{(t)}$, ${}_tm_x^{(t)}$ (t), be able to: • Establish relations between the functions • Given the joint distribution of the time of failure and the cause of failure, calculate probabilities and moments associated with functions of these random variables • Apply assumptions about the pattern of failures between integral ages to obtain the associated (discrete) single decrement models from a discrete multiple decrement model as well as the discrete multiple decrement model that results from two or more discrete	f. Time until failure g. Competing risk (multiple decrement) models

single decrement models *Range of weight: 3-7 percent*	
READINGS	
Bowers et al.	

B. **Frequency and Severity Models**
Range of weight for Section B: 25-30 percent

Candidates should be able to define frequency (counting) and severity distributions, and be able to use the parameters and moments of these distributions. Candidates also should be able to work with the families of distributions generated by algebraic manipulation and mixing of the basic distributions presented.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
6. For the following counting distribution (frequency distribution): Poisson, mixed Poisson, negative binomial, binomial, and the (a,b,1) class of distributions, be able to: • Describe the how changes in the parameters values impact the distribution • Calculate their moments • Identify the applications for which these distributions are used and the reasons why they are used • Given the parameters of a distribution, apply the distribution to an application *Range of weight: 8-12 percent*	f. Applications of Frequency Distributions g. Parameters of Frequency Distribution h. Moments of Frequency Distributions
2. For the following families of loss (severity) distributions transformed beta, transformed gamma, inverse transformed gamma, lognormal and inverse Gaussian: • Describe how changes in the parameters values affect the distribution • Calculate their moments • Apply the following techniques for creating new families of distributions: multiplication by a constant, raising to a power, exponentiation, and mixing • Identify the applications in which these distributions are used and the reasons why they are used • Given the parameters of a	b. Applications of Loss Distributions c. Parameters of Loss Distribution d. Moments of Loss Distributions e. Creation of new distributions

distribution, apply the distribution to an application *Range of weight: 8-12 percent*	
3. Be able to interpret and produce graphical representations of loss and counting distributions. Be able to identify graphical presentations of loss that are: • Eliminated by a deductible • Covered under an insurance contract • Excess of the coverage provided by an insurance contract *Range of weight: 3-7 percent*	c. Lee diagrams
READINGS	
Klugman SN Klugman et al. 1 Lee	

B. Compound Distribution Models

Range of weight for Section C: 5-10 percent

Candidates should be able to calculate the probabilities associated with a compound distribution when the compounding distribution is one of the frequency distributions presented in the syllabus, and the compounded distribution is discrete or a discretization of a continuous distribution. Candidates also should be able to adjust such probability calculations for the impact of policy modifications such as deductibles, policy limits, and coinsurance.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
3. Describe a compound distribution. *Range of weight: 0-5 percent*	c. Compound distributions
2. Calculate probabilities associated with a compound distribution when the compounding distribution is a member of the families in Learning Objective B1, and the compounded distribution is discrete or a discretization of a continuous distribution. *Range of weight: 0-5 percent*	b. Probabilities implied by compound distributions
3. Adjust the calculations described in Learning Objective C2 for the impact of policy modifications such as deductibles, policy limits and coinsurance. *Range of weight: 0-5 percent*	c. Impact of deductible, policy limits and coinsurance
READINGS	
Klugman et al. 2	

B. **Stochastic Process Models**

Range of weight for Section D: 20-25 percent

Candidates should learn to solve problems using stochastic processes. They also should learn how to determine the probabilities and distributions associated with these processes.

The following stochastic processes will be covered: Markov chain (discrete-time and continuous-time) processes, counting processes, Poisson process (including nonhomogeneous and compound Poisson processes), and Brownian motion.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
3. For stochastic process, describe a process and be able to distinguish between discrete-time and continuous-time processes. *Range of weight: 3-7 percent*	c. Stochastic process d. Discrete time process e. Continuous time process
2. Describe a discrete-time Markov chain in terms of the transition probability matrix. • Use the Chapman-Kolmogorov equations to obtain probabilities associated with a discrete-time Markov chain. • Classify the states of a discrete-time Markov chain. • Calculate the limiting probabilities of a discrete-time Markov chain. *Range of weight: 3-7 percent*	c. Markov chains d. Transition probability matrix e. Discrete-time Markov chains
3. Describe a counting process. *Range of weight: 0-5 percent*	c. Counting process
4. For a Poisson process be able to calculate: • The distribution of the waiting times between events • The distribution of the process increments • The behavior of the process over an infinitesimal time interval *Range of weight: 0-5 percent*	d. Poisson process
5. Describe a nonhomogeneous Poisson process. For this process, be able to calculate probabilities	e. Nonhomogeneous Poisson process

associated with numbers of events and time periods of interest. *Range of weight: 0-5 percent*	f. Probability calculations for Nonhomogeneous Poisson process
6. For a compound Poisson process: • Calculate moments associated with the value of the process at a given time • Describe the value of the process at a given time as a compound Poisson random variable *Range of weight: 0-5 percent*	f. Compound Poisson process
7. Describe a Brownian motion process and be able to: • Determine the distribution of the value of the process at any time • Determine the distribution of a hitting time • Calculate the probability that one hitting time will be smaller than another • Describe a Brownian motion process with drift and a geometric Brownian motion process *Range of weight: 0-5 percent*	g. Brownian motion process h. Hitting times i. Brownian motion process with drift j. Geometric Brownian motion process
READINGS	
Ross 1	

B. **Ruin Models**

Range of weight for Section E: 5-10 percent

Candidates should be able to analyze the probability of ruin using various models. Other topics covered in this section include the determination of the characteristics of the distribution of the amount of surplus (deficit) at the first time below the initial level and the impact of reinsurance. (Knowledge regarding reinsurance terminology is not assumed. Cash flows from reinsurance will be determinable based on the description of the reinsurance provided on the examination.)

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
7. For a ruin model: • Describe the considerations	g. Ruin models

included in a ruin model • Calculate ruin probabilities for discrete time surplus processes *Range of weight: 5-10 percent*	
READINGS	
Klugman et al. 3	

A. **Simulation Modeling**
Range of weight for Section F: 5-10 percent

Candidates should be able to generate discrete and continuous random variables using basic simulation methods. They also should be able to construct algorithms to simulate outcomes using stochastic models.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
7. Generate discrete and continuous random variables using basic simulation methods. *Range of weight: 3-7 percent*	. Simulation basics a. Applications to generate values of discrete and continuous random variables
2. Construct an algorithm to appropriately simulate outcomes under a wide variety of stochastic models. *Range of weight: 0-5 percent*	b. Simulation algorithms
READINGS	
Ross 2	

Complete Text References for Exam 3

Text references are alphabetized by the citation column.

Citation	Abbreviation	Learning Objectives	Source
Bowers, N.L.; Gerber, H.U.; Hickman, J.C.; Jones, D.A.; and Nesbitt, C.J., *Actuarial Mathematics* (Second Edition), 1997, Society of Actuaries, Chapter 3 (excluding 3.6), Sections 4.1-4.3, 5.1-5.3, 6.1-6.3, 7.1-7.4, 9.1-9.5, 9.7, 10.1-10.3.	Bowers et al.	A1-A6	L
Klugman, S.A., "Course/Exam 3 Study Note Replacing Chapter 2 Material from Loss Models," (Please use the fourth printing, December 1, 2003.) June 2003.	Klugman SN	B1-B3	W
Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions*, 1998, John Wiley and Sons, New York, Sections 1.3, 3.1, 3.2.1-3.2.2, 3.3.1-3.3.2, 3.4.1, 3.5 (through first full paragraph on p. 222), 3.7 (excluding Examples 3.15, Theorem 3.4, Example 3.18 and following), 3.10.1 (excluding Example 3.34 and following), 3.10.2 (excluding Example 3.38 and following). [Some notation used in *Loss Models: From Data to Decisions* is introduced in Section 3.6.1. The candidate may find it helpful to refer to Section 3.6.1 when studying the later sections of the text.]	Klugman et al. 1	B1-B3	L

Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions*, 1998, John Wiley and Sons, New York, Sections 1.3, 3.1, 3.2.1-3.2.2, 3.3.1-3.3.2, 3.4.1, 3.5 (through first full paragraph on p. 222), 3.7 (excluding Examples 3.15, Theorem 3.4, Example 3.18 and following), 3.10.1 (excluding Example 3.34 and following), 3.10.2 (excluding Example 3.38 and following). [Some notation used in *Loss Models: From Data to Decisions* is introduced in Section 3.6.1. The candidate may find it helpful to refer to Section 3.6.1 when studying the later sections of the text.]	Klugman et al. 1	B1-B3	**L**
Klugman et al., Loss Models: From Data to Decisions, 1998, Sections 1.4, 4.1-4.3, 4.5, 4.6 (excluding Theorem 4.4 and Sections 4.6.2-4.6.5), 4.8.	Klugman et al. 2	C1-C3	**L**
Klugman et al., *Loss Models: From Data to Decisions*, 1998, Sections 6.2.3, 6.3.1, 6.3.2.1.	Klugman et al. 3	E	**L**
Lee, Y.S., "The Mathematics of Excess of Loss Coverages and Retrospective Rating-A Graphical Approach," Section 1, *PCAS* LXXV, 1988, pp. 49-54.	Lee	B1-B3	**W**
Ross, S.M., *Introduction to Probability Models* (Eighth Edition), 2003, Academic Press, San Diego, Sections 2.8, 4.1-4.4, 4.5.1, 4.6, 5.3-5.4 (excluding 5.4.3), 10.1-10.3. [Candidates may also use the seventh edition with the following citation: Sections 2.8, 4.1-4.4, 4.5.1, 4.6, 5.3-5.4, 10.1-10.3.]	Ross 1	D1-D7	**L**
Ross, S.M., *Simulation* (Third Edition), 2002, Academic Press, San Diego, Sections 3.1, 4.1-4.3, Chapter 5 (excluding 5.3 and 5.5). [Candidates may also use the Second Edition, 1997. The same chapter and section references apply.]	Ross 2	F1-F2	**L**

Key

L	May be purchased from the publisher or bookstore or borrowed from the CAS Library.
NEW	Indicates new or updated material or modified citation.
W	Represents material that is available free-of-charge from the CAS Web Site.

Publishers and Distributors

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Actuarial Bookstore, P.O. Box 69, Greenland, NH 03840; telephone: (800) 582-9672 (U.S. only) or (603) 430-1252; fax: (603) 430-1258; Web site: www.actuarialbookstore.com.

Bowers, N.L.; Gerber, H.U.; Hickman, J.C.; Jones, D.A.; and Nesbitt, C.J., *Actuarial Mathematics* (Second Edition), 1997, Society of Actuaries, 475 N. Martingale Road, Suite 600, Schaumburg, IL 60173-2226; telephone: (847) 706-3500; fax: (847) 706-3599; Web site: www.soa.org.

Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions*, 1998, John Wiley and Sons, One Wiley Drive, Somerset, NJ 08875; telephone: (800) 225-5945 or (732) 469-4400.

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Ross, S.M., *Introduction to Probability Models* (Eighth Edition), 2003, Academic Press, 6277 Sea Harbor Drive, Attn: Customer Service (Fifth Floor), Orlando, FL 32887; telephone: (407) 345-3800.
Ross, S.M., *Simulation* (Third Edition), 2002, Academic Press, 6277 Sea Harbor Drive, Attn: Customer Service (Fifth Floor), Orlando, FL 32887; telephone: (407) 345-3800.
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