SOA/CAS Course 4 - Actuarial Modeling

This four-hour multiple-choice examination is administered by Preliminary Actuarial Examinations and is identical to SOA Course 4. Items marked with a bold W are available at no charge under Web Notes.

Please check the Admissions section for any changes to the *Syllabus*.

This examination provides an introduction to modeling and covers important actuarial and statistical methods that are useful in modeling. A thorough knowledge of calculus, linear algebra, probability, and mathematical statistics is assumed. The candidate will be required to understand the steps involved in the modeling process and how to carry out these steps in solving business problems. The candidate should be able to: 1) analyze data from an application in a business context; 2) determine a suitable model including parameter values; and 3) provide measures of confidence for decisions based upon the model. The candidate will be introduced to a variety of tools for the calibration and evaluation of the models on Exam 3.

A variety of tables will be provided to the candidate in the Study Note Package and in the examination booklet. These include values for the standard normal distribution, chi-square distribution, *t* distribution, *F* distribution, and abridged inventories of discrete and continuous probability distributions. Since they will be included with the examination, candidates will not be allowed to bring copies of the tables into the examination room.

Learning Objectives

Understanding Actuarial Models
The candidate is expected to apply statistical methods to sample data to quantify and evaluate the models presented on Exam 3 and to use the models to solve problems set in a business context. The effects of regulations, laws, accounting practices, and competition on the results produced by these models are not considered in this exam.

The candidate is expected to be able to perform the tasks listed below.

1. Identify the steps in the modeling process and discuss how they interrelate.

2. Identify the models and methods available, and understand the difference between the models and the methods.

3. Explain the difference between a stochastic and a deterministic model and identify the advantages and disadvantages of each.

4. Discuss the possible limitations imposed by the data available for input for constructing a model.

5. Understand that all models presented in Exams 3 and 4 are closely related. Apply models from more than one family (e.g., regression, stochastic process, time series) to a particular business application.

6. Identify the underlying assumptions implicit in each family of models and recognize which set(s) of assumptions are applicable to a given business application.

7. Estimate the parameters of a tabular failure time or loss distribution when the data is complete, or when it is incomplete, using maximum likelihood, method of moments, and Bayesian estimation.

8. Obtain nonparametric estimates for a failure time or loss distribution using the empirical distribution, the Kaplan-Meier estimator, and the Nelson-Aalen estimator.

9. Construct the likelihood model needed to estimate the parameters of a parametric failure time or loss distribution regression model.

10. Construct the partial likelihood model needed to estimate the regression coefficients in a semiparametric failure time or loss distribution regression model.

11. Adjust an estimation based on the presentation of the sample data-complete, incomplete, censored, truncated, grouped, shifted.

12. Apply statistical tests to determine the acceptability of a fitted model:

    - Pearson's chi-square statistic
    - Likelihood ratio test
    - Kolmogorov-Smirnov statistic

13. For estimators, define the terms: efficiency, bias, consistency, and mean squared error.

14. Calculate the least squares estimates of the parameters used in single and multiple linear regression models, and use knowledge of their distributions for hypothesis testing and development of confidence intervals.

15. Test a given linear regression model's fit to a given data set.

16. Assess the appropriateness of the linear regression model for a given data set by checking for such irregularities as heteroscedasticity, serial correlation, and multicollinearity.

17. Develop deterministic forecasts from time series data, using simple extrapolation and moving average models, applying smoothing techniques and seasonal adjustment when appropriate.

18. Use the concept of the autocorrelation function of a stochastic process to test the process for stationarity.

19. Generate a forecast using the general ARIMA model and develop confidence intervals for the forecast.

20. Test the hypothesis that a given stochastic process is a random walk.

21. For an ARIMA process (including simpler models as special cases), estimate the model parameters, and perform appropriate diagnostic checks of the model.

22. Apply limited fluctuation (classical) credibility including criteria for both full and partial credibility.

23. Perform Bayesian analysis using discrete and continuous examples.

24. Apply the Buhlmann-Straub credibility model to basic situations. Understand the relationship to the Bayesian model.

25. Apply the conjugate prior in Bayesian analysis and Buhlmann-Straub credibility, and, in particular, to the Poisson-gamma model.

26. Apply empirical Bayesian methods in the nonparametric and semiparametric cases.

27. Compare and contrast the assumptions underlying limited fluctuation credibility, Bayesian analysis, and the Buhlmann-Straub credibility model.

28. Determine an appropriate number of simulations to perform in order to estimate a quantity of interest.

29. Quantify the variability of an estimate in the context of simulation.

30. Determine the bootstrap estimates of the mean squared error of an estimator.

31. Use basic simulation methods to validate a model.

Applications of Actuarial Models
The candidate is expected to apply the models presented in Exam 3 and the statistical methods presented on this exam to business applications. As discussed above, the candidate should be able to take data from a given application and determine a suitable model, including parameter estimates, for use in making business decisions related to the application. The candidate should be able to assess the variability of the parameter estimates and the goodness of fit of the model, and therefore provide an opinion on the confidence that should be given to the model output in making decisions. Relevant business applications include, but are not limited to:

- Premium (rate) for life insurance and annuity contracts
- Premium (rate) for accident and health insurance contracts
- Premium (rate) for casualty (liability) insurance contracts
- Premium (rate) for property insurance contracts
- Rates for coverages under group benefit plans
- Loss reserves for insurance contracts
- Benefit reserves for insurance contracts
- Resident fees for Continuing Care Retirement Communities (CCRCs)
- Cost of a warranty for manufactured goods
- Value of a financial instrument such as: a loan, a stock, an option, etc.
- Risk classification

Note: Concepts, principles, and techniques needed for Exam 4 are covered in the references listed below. Candidates and professional educators may use other references, but candidates should be very familiar with the notation and terminology used in the listed references.

A. The Modeling Process
Candidates should be able to identify steps in the modeling process as well as understand specific methods, models, underlying assumptions, and limitations imposed by the data.

READINGS

Background reading: Jones, B.L., "Actuarial Models and Modeling: An Interactive Approach" (CD-ROM), 2000, ACTEX Publications. (This reference is not required but may be a valuable tool to explore actuarial models and modeling techniques relevant to this exam.)

B. Estimation and Fitting of Models
Candidates should be able to construct models and estimate model parameters using the models and methods contained in the readings. Sample data used for estimation may be complete, incomplete, censored, truncated, grouped, or shifted. Ability to apply tests to determine the acceptability of a model will also be required.

READINGS

W Klugman, S.A., "Estimation, Evaluation, and Selection of Actuarial Models,"(Please use the third printing, December 1, 2003.) Study Note, June 2003.

C. Regression, Forecasting, and Time Series

Candidates should be able to understand the basics of regression analysis, time series analysis, and forecasting. Candidates will be required to estimate model parameters, perform various tests of the model to determine its acceptability, and generate forecasts using the model (with a confidence interval).

READINGS

Pindyck, R.S.; and Rubinfeld, D.L., *Econometric Models and Economic Forecasts* (Fourth Edition), 1998, Irwin McGraw-Hill, Boston, Chapters 3-6, 15-18.

D. Credibility Theory

Candidates should have a thorough understanding of credibility theory and concepts contained in the readings. Knowledge of limited fluctuation credibility, Bayesian and empirical Bayesian methods, Bulhmann and Buhlmann-Straub credibility is required.

BACKGROUND READINGS

Before commencing formal study of the material in this section, candidates should read the following for an introduction to the basic ideas underlying credibility theory:

W Philbrick, S.W., "An Examination of Credibility Concepts," *PCAS* LXVII, 1981, pp. 195-212.

Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions*, 1998, John Wiley and Sons, New York, Sections 1.5 and 5.1.

W Mahler, H.C.; and Dean, C.G., "Credibility," *Foundations of Casualty Actuarial Science* Science (Fourth Edition), 2001, Casualty Actuarial Society, Chapter 8, Section 1.

In addition, Section 5.2 of *Loss Models: From Data to Decisions* by Klugman, Panjer, and Willmot contains a review of basic statistical concepts that some candidates may find useful.

READINGS

W Mahler, H.C.; and Dean, C.G., "Credibility," *Foundations of Casualty Actuarial Science* (Fourth Edition), 2001, Casualty Actuarial Society, Chapter 8, Section 2.

The candidate may use either course of reading (Option 1 or Option 2) listed below for the remainder of the credibility material. The candidate will not be tested on the details of derivations in either course of reading.

Option 1

Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions*, 1998, John Wiley and Sons, New York, Sections 5.4 and 5.5 (excluding 5.4.6 and 5.5.3).

Option 2

W Mahler, H.C.; and Dean, C.G., "Credibility," *Foundations of Casualty Actuarial Science* (Fourth Edition), 2001, Casualty Actuarial Society, Chapter 8, Section 3-5; and

Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions*, 1998, John Wiley and Sons, New York, Sections 5.4.4. and 5.5 (excluding 5.5.3).

E. Simulation in Estimation and Fitting

Candidates should be able to apply simulation methods as presented in the readings to areas such as estimating a quantity, determining an estimate's variability, and validating a model.

READINGS

Ross, S.M., *Simulation* (Third Edition), 2002, Academic Press, San Diego, Chapters 7 and 9 (excluding 9.4). [Candidates may also use the Second Edition, 1997. The same chapter and section references apply.]

Key

L	May be borrowed from the CAS Library.

NEW	Indicates new or updated material or modified citation.
SK	Represents material included in the 2003 CAS Study Kit.
SKU	Represents material included in the 2003 CAS Study Kit *and* the 2003 Update to the 2002 Study Kit
W	Represents material that is available free-of-charge from the CAS Web Site.

Publishers and Distributors

Contact information is furnished for those who wish to purchase the text references cited for Exam 4. Publishers and distributors are independent and listed for the convenience of candidates; inclusion does not constitute endorsement by the CAS.

ACTEX Publications, 140 Willow Street, Suite One, P.O. Box 974, Winsted, CT 06098; telephone: (800) 282-2839 or (860) 379-5470; fax: (860) 738-3152, Web site: www.actexmadriver.com; e-mail: retail@actexmadriver.com.

Actuarial Bookstore, P.O. Box 69, Greenland, NH 03840; telephone: (800) 582-9672 (US only) or (603) 430-1252; fax: (603) 430-1258; Web site: www.actuarialbookstore.com.

Casualty Actuarial Society *Forum*, *Foundations of Casualty Actuarial Science* (Fourth Edition), *PCAS*, and *Discussion Paper Program*, 1100 N. Glebe Road, Suite 600, Arlington, VA 22201-4798; telephone: (703) 276-3100; fax: (703) 276-3108; e-mail: office@casact.org

Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions*, 1998, John Wiley and Sons, One Wiley Drive, Somerset, NJ 08875; telephone: (800) 225-5945 or (732) 469-4400.

Pindyck, R.S.; and Rubinfeld, D.L., *Econometric Models and Economic Forecasts* (Fourth Edition), 1998, Irwin McGraw-Hill, P.O. Box 182605, Columbus, OH 43218-2605; telephone: (800) 262-4729.

Ross, S.M., *Simulation* (Third Edition), 2002, Academic Press, 6277 Sea Harbor Drive, Attn: Customer Service (Fifth Floor), Orlando, FL 32887; telephone: (407) 345-3800.

SlideRule Books, 10 First Avenue East, Mobridge, SD 57601; telephone: (877) 407-5433 or (605) 845-5580; fax: (877) 417-5433 or (605) 845-7627; Web site: www.sliderulebooks.com.