SOA Course 3 – Actuarial Models

The examination for this course consists of four hours of multiple-choice questions.

This course develops the candidate's knowledge of the theoretical basis of actuarial models and the application of those models to insurance and other financial risks. A thorough knowledge of calculus, probability and interest theory is assumed. Knowledge of risk management at the level of Course 1 is also assumed.

The candidate will be required to understand, in an actuarial context, what is meant by the word "model," how and why models are used, their advantages and their limitations. The candidate will be expected to understand what important results can be obtained from these models for the purpose of making business decisions, and what approaches can be used to determine these results.

A variety of tables will be provided to the candidate in the study note package and at the examination. These include values for the standard normal distribution, illustrative life tables, and abridged inventories of discrete and continuous probability distributions. These tables are also available on the SOA and CAS Web sites. Since they will be included with the examination, candidates will not be allowed to bring copies of the tables into the examination room.

Learning Objectives

Understanding Actuarial Models

The candidate is expected to understand the models and techniques listed below and to be able to apply them to solve problems set in a business context. The effects of regulations, laws, accounting practices and competition on the results produced by these models are not considered in this course. The candidate is expected to be able to:

1. Explain what a mathematical model is and, in particular, what an actuarial model can be.

2. Discuss the value of building models for such purposes as: forecasting, estimating the impact of making changes to the modeled situation, estimating the impact of external changes on the modeled situation.

3. Identify the models and methods available, and understand the difference between the models and the methods.

4. Explain the difference between a stochastic and a deterministic model and identify the advantages/disadvantages of each.

5. Understand that all models presented (e.g., survival models, stochastic processes, aggregate loss models) are closely related.

6. Formulate a model for the present value, with respect to an assumed interest rate structure, of a set of future contingent cash flows. The model may be stochastic or deterministic.

7. Determine the characteristics of the components and the effects of changes to the components of the model in 6. Components include:
   - a deterministic interest rate structure;
   - a scheme for the amounts of the cash flows;
   - a probability distribution of the times of the cash flows; and
   - the probability distribution of the present value of the set of cash flows.
8. Apply a principle to a present value model to associate a cost or pattern of costs (possibly contingent) with a set of future contingent cash flows.
   - Principles include: equivalence, exponential, standard deviation, variance, and percentile.
   - Models include: present value models based on 9-12 below.
   - Applications include: insurance, health care, credit risk, environmental risk, consumer behavior (e.g., subscriptions), and warranties.
9. Characterize discrete and continuous univariate probability distributions for failure time random variables in terms of the life table functions, l_x, q_x, p_x, ${}_nq_x$, ${}_np_x$, and ${}_{m|n}q_x$, the cumulative distribution function, the survival function, the probability density function and the hazard function (force of mortality), as appropriate.
   - Establish relations between the different functions.
   - Develop expressions, including recursion relations, in terms of the functions for probabilities and moments associated with functions of failure time random variables, and calculate such quantities using simple failure time distributions.
   - Express the impact of explanatory variables on a failure time distribution in terms of proportional hazards and accelerated failure time models.
10. Given the joint distribution of two failure times:
    - Calculate probabilities and moments associated with functions of these random variables.
    - Characterize the distribution of the smaller failure time (the joint life status) and the larger failure time (the last survivor status) in terms of functions analogous to those in 9, as appropriate.
    - Develop expressions, including recursion relations, for probabilities and moments of functions of the joint life status and the last survivor status, and express these in terms of the univariate functions in 9 in the case in which the two failure times are independent.
    - Characterize the joint distribution of two failure times, the joint life status and the last survivor status using the common shock model.

11. Characterize the joint distribution (pdf and cdf) of the time until failure and the cause of failure in the competing risk (multiple decrement) model, in terms of the functions
$l_x^{(\tau)}, {}_tq_x^{(\tau)}, {}_tp_x^{(\tau)}, {}_td_x^{(\tau)}, {}_t\mu_x^{(\tau)}(t)$

    - Establish relations between the functions.

    - Given the joint distribution of the time of failure and the cause of failure, calculate probabilities and moments associated with functions of these random variables.

    - Apply assumptions about the pattern of failures between integral ages to obtain the associated (discrete) single decrement models from a discrete multiple decrement model as well as the discrete multiple decrement model that results from two or more discrete single decrement models.

12. Generalize the models of 9, 10, and 11 to multiple state models characterized in terms of transition probability functions or transition intensity functions (forces of transition).

13. Define a counting distribution (frequency distribution).

    - Characterize the following distributions in terms of their parameters and moments: Poisson, mixed Poisson, negative binomial, and binomial distributions.

    - Identify the applications for which these distributions are used and the reasons why they are used.

    - Given the parameters of a distribution, apply the distribution to an application.

14. Define a loss distribution.

    - Characterize the following families of distributions in terms of their parameters and moments: transformed beta, transformed gamma, inverse transformed gamma, lognormal and inverse Gaussian.

    - Apply the following techniques for creating new families of distributions: multiplication by a constant, raising to a power, exponentiation, and mixing.

    - Identify the applications in which these distributions are used and the reasons why they are used.

    - Given the parameters of a distribution, apply the distribution to an application.

15. Define a compound distribution.

16. Calculate probabilities associated with a compound distribution when the compounding distribution is a member of the families in 13, and the

compounded distribution is discrete or a discretization of a continuous distribution.

17. Adjust the calculation of 16 for the impact of policy modifications such as deductibles, policy limits and coinsurance.
18. Define a stochastic process and distinguish between discrete-time and continuous-time processes.
19. Characterize a discrete-time Markov chain in terms of the transition probability matrix.
    - Use the Chapman-Kolmogorov equations to obtain probabilities associated with a discrete-time Markov chain.
    - Classify the states of a discrete-time Markov chain.
    - Calculate the limiting probabilities of a discrete-time Markov chain.
20. Define a counting process.
21. Characterize a Poisson process in terms of:
    - the distribution of the waiting times between events,
    - the distribution of the process increments,
    - the behavior of the process over an infinitesimal time interval.
22. Define a nonhomogeneous Poisson process.
    - Calculate probabilities associated with numbers of events and time periods of interest.
23. Define a compound Poisson process.
    - Calculate moments associated with the value of the process at a given time.
    - Characterize the value of the process at a given time as a compound Poisson random variable.
24. Define a Brownian motion process.
    - Determine the distribution of the value of the process at any time.
    - Determine the distribution of a hitting time.
    - Calculate the probability that one hitting time will be smaller than another.
    - Define a Brownian motion process with drift and a geometric Brownian motion process.
25. For a discrete-time surplus process:

- Calculate the probability of ruin within a finite time by a recursion relation.
- Analyze the probability of ultimate ruin via the adjustment coefficient and establish bounds.

26. For a continuous-time Poisson surplus process:
    - Derive an expression for the probability of ruin assuming that the claim amounts are combinations of exponential random variables.
    - Calculate the probability that the surplus falls below its initial level, determine the deficit at the time this first occurs, and characterize the maximal aggregate loss as a compound geometric random variable.
    - Approximate the probability of ruin using the compound geometric recursion.
    - Analyze the probability of ruin: analytically (e.g., adjustment coefficient); numerically; and by establishing bounds.
    - Determine the characteristics of the distribution of the amount of surplus (deficit) at: first time below the initial level; and the lowest level (maximal aggregate loss).
27. Analyze the impact of reinsurance on the probability of ruin and the expected maximum aggregate loss of a surplus process.
28. Generate discrete random variables using basic simulation methods.
29. Generate continuous random variables using basic simulation methods.
30. Construct an algorithm to appropriately simulate outcomes under a stochastic model.

Applications of Actuarial Models

The candidate is expected to be able to apply the models above to business applications. The candidate should be able to determine an appropriate model for a given business problem and be able to determine quantities that are important in making business decisions, given the values of the model parameters. Relevant business applications include, but are not limited to:

- Premium (rate) for life insurance and annuity contracts,
- Premium (rate) for accident and health insurance contracts,
- Premium (rate) for casualty (liability) insurance contracts,
- Premium (rate) for property insurance contracts,
- Rates for coverages under group benefit plans,

- Loss reserves for insurance contracts,
- Benefit reserves for insurance contracts,
- Resident fees for Continuing Care Retirement Communities (CCRCs),
- Cost of a warranty for manufactured goods,
- Value of a financial instrument such as: a loan, a stock, an option, etc.,
- Risk classification,
- Solvency (ruin).

Note: Concepts, principles and techniques needed for Course 3 are covered in the references listed below. Candidates and professional educators may use other references, but candidates should be very familiar with the notation and terminology used in the listed references.

Texts

- *Actuarial Mathematics* (Second Edition), 1997, by Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.A. and Nesbitt, C.J., Chapters 3-4, Sections 5.1-5.4, 6.1-6.4, 7.1-7.6, 8.1-8.4, 9.1-9.5, 9.6 (excluding 9.6.2), 9.7-9.8, Chapter 10 (excluding 10.5.2 and 10.5.5), Sections 11.1-11.3 and Chapter 13 (excluding autoregressive discrete-time model and appendix).
  Note: Some notation presented in Chapter 13 of *Actuarial Mathematics* is introduced in Chapter 12. Candidates may find it helpful to refer to Chapter 12 when studying the readings in Chapters 13.

- *Introduction to Probability Models* (Eighth Edition), 2003, by Ross, S.M., Sections 2.8, 4.1-4.4, 4.5.1, 4.6, 5.3-5.4, 10.1-10.3. (Candidates may also use the Seventh Edition – same sections)

- *Loss Models: From Data to Decisions*, 1998, by Klugman, S.A., Panjer, H.H., and Willmot, G.E., Sections 1.3, 1.4, 3.1, 3.2.1-3.2.2, 3.3.1-3.3.2, 3.4.1, 3.5 (through first full paragraph on page 222), 3.7 (excluding Example 3.15, Theorem 3.4, Example 3.18 and following), 3.10.1 (excluding Example 3.34 and following), 3.10.2 (excluding Example 3.38 and following), 4.1-4.3, 4.5, 4.6 (excluding Theorem 4.4 and Sections 4.6.2-4.6.5), 4.8, 6.2.3, 6.3.1, 6.3.2.1.
  Note: Some notation presented in *Loss Models: From Data to Decisions* is introduced in Section 3.6.1. The candidate may find it helpful to refer to Section 3.6.1 when studying the later sections of the text.

- *Simulation* (Third Edition), 2002, by Ross, S.M., Sections 3.1, 4.1-4.3, Chapter 5 (excluding 5.3 and 5.5). [Candidates may also use the Second Edition, 1997. The same chapter and section references apply.]