

SUBJECT 102: FINANCIAL MATHEMATICS

Aim

The aim of the Financial Mathematics course is to provide grounding in financial mathematics and their simple applications.

Objectives

On completion of the course the trainee actuary will be able to:

(i) Describe how to use a generalised cash-flow model to describe financial transactions, making allowance for the probability of payment.

1. For a given cash flow process, state the inflows and outflows in each future time period and discuss whether the amount or the timing (or both) is fixed or uncertain.
2. Describe in the form of a cash flow model the operation of a zero coupon bond, a fixed interest security, an index-linked security, cash on deposit, an equity, an “interest only” loan, a repayment loan, an annuity, a temporary assurance, an endowment assurance and a motor vehicle insurance policy.

(ii) Describe how to take into account the time value of money using the concepts of compound interest and discounting.

1. Accumulate a single investment at a constant rate of interest under the operation of:
 - simple interest
 - compound interest
2. Define the present value of a future payment.
3. Discount a single investment under the operation of simple (commercial) discount at a constant rate of discount.
4. Describe how a compound interest model can be used to represent the effect of investing a sum of money over a period.

(iii) Show how interest rates or discount rates may be expressed in terms of different time periods.

1. Derive the relationship between the rates of interest and discount over one effective period arithmetically and by general reasoning.
2. Derive the relationships between the rate of interest payable once per effective period and the rate of interest payable p times per time period and the force of interest.
3. Explain the difference between nominal and effective rates of interest and derive effective rates from nominal rates.
4. Calculate the equivalent annual rate of interest implied by the accumulation of a sum of money over a specified period where the force of interest is a function of time.

(iv) Calculate the present value and the accumulated value of a stream of equal or unequal payments using specified rates of interest and the net present value at a real rate of interest, assuming a constant rate of inflation.

1. Discount and accumulate a sum of money or a series (possibly infinite) of cash flows to any point in time where:
 - the rate of interest or discount is constant
 - the rate of interest or discount varies with time but is not a continuous function of time
 - either or both the rate of cash flow and the force of interest are continuous functions of time

2. Calculate the present value and accumulated value of a series of equal or unequal payments made at regular intervals under the operation of specified rates of interest where the first payment is:

- deferred for a period of time
- not deferred

(v) Define and use the more important compound interest functions.

(vi) Define an equation of value.

1. Define an equation of value, where payment or receipt is certain.
2. Describe how an equation of value can be adjusted to allow for uncertain receipts or payments.

(vii) Describe how a loan may be repaid by regular instalments of interest and capital.

1. Describe flat rates and annual effective rates.
2. Calculate a schedule of repayments under a loan and identify the interest and capital components of annuity payments where the annuity is used to repay a loan for the case where annuity payments are made once per effective time period or p times per effective time period and identify the capital outstanding at any time.

(viii) Show how discounted cash flow techniques can be used in investment project appraisal.

1. Calculate the net present value and accumulated profit of the receipts and payments from an investment project at given rates of interest.
2. Calculate the internal rate of return implied by the receipts and payments from an investment project.
3. Describe payback period and discounted payback period and discuss their suitability for assessing the suitability of an investment project.
4. Determine the payback period and discounted payback period implied by the receipts and payments from an investment project.
5. Calculate the money-weighted rate of return, the time-weighted rate of return and the linked internal rate of return on an investment or a fund.

(ix) Describe the investment and risk characteristics of the following types of asset available for investment purposes:

- fixed-interest Government borrowings
- fixed-interest borrowing by other bodies
- shares and other equity-type finance
- derivatives

(x) Analyse simple compound interest problems.

1. Calculate the present value of payments from a fixed interest security where the coupon rate is constant and the security is redeemed in one instalment.
2. Calculate upper and lower bounds for the present value of a fixed interest security that is redeemable on a single date within a given range at the option of the borrower.
3. Calculate the running yield and the redemption yield from a fixed interest security (as in 1.), given the price.
4. Calculate the present value or yield from an ordinary share and a property, given simple (but not necessarily constant) assumptions about the growth of dividends and rents.

5. Solve an equation of value for the real rate of interest implied by the equation in the presence of specified inflationary growth.
6. Calculate the present value or real yield from an index-linked bond, given assumptions about the rate of inflation.
7. Calculate the price of, or yield from, a fixed interest security where the investor is subject to deduction of income tax on coupon payments and redemption payments are subject to the deduction of capital gains tax.
8. Calculate the value of an investment where capital gains tax is payable, in simple situations, where the rate of tax is constant, indexation allowance is taken into account using specified index movements and allowance is made for the case where an investor can offset capital losses against capital gains.

(xi) Calculate the delivery price and the value of a forward contract using arbitrage free pricing methods.

1. Define “arbitrage” and explain why arbitrage may be considered impossible in many markets.
2. Calculate the price of a forward contract assuming arbitrage is impossible, and assuming that there is no income or expenditure associated with the underlying asset during the term of the contract.
3. Explain what is meant by “hedging” in the case of a forward contract.
4. Calculate the value of a forward contract at any time during the term of the contract, assuming:
 - no income or expenditure associated with the underlying asset during the term of the contract
 - a fixed income from the asset during the term
 - a fixed dividend yield from the asset during the term

(xii) Show an understanding of the term structure of interest rates.

1. Describe the main factors influencing the term structure of interest rates.
2. Explain what is meant by the par yield and yield to maturity.
3. Explain what is meant by, derive the relationships between and evaluate:
 - discrete spot rates and forward rates
 - continuous spot rates and forward rates
4. Define the duration and convexity of a cash flow sequence, and illustrate how these may be used to estimate the sensitivity of the value of the cash flow sequence to a shift in interest rates.
5. Evaluate the duration and convexity of a cash flow sequence.
6. Explain how duration and convexity are used in the (Redington) immunisation of a portfolio of liabilities.

(xiii) Show an understanding of simple stochastic interest rate models.

1. Describe the concept of a stochastic interest rate model and the fundamental distinction between this and a deterministic model.
2. Derive algebraically, for the model in which the annual rates of return are independently and identically distributed and for other simple models, expressions for the mean value and the variance of the accumulated amount of a single premium.
3. Derive algebraically, for the model in which the annual rates of return are independently and identically distributed, recursive relationships which permit the evaluation of the mean value and the variance of the accumulated amount of an annual premium.

4. Derive analytically, for the model in which each year the random variable $(1 + i)$ has an independent log-normal distribution, the distribution functions for the accumulated amount of a single premium and for the present value of a sum due at a given specified future time.
5. Apply the above results to the calculation of the probability that a simple sequence of payments will accumulate to a given amount at a specific future time.

The Main Reading: UK Institute of Actuaries Core Reading for subject 102 Financial Mathematics

Additional Reading:

1. McCutcheon, John J. and Scott, William F., *An introduction to the mathematics of finance*. Butterworth-Heinemann, 2002, ISBN 0750600926
2. S. Broverman, *Mathematics of Investment and Credit*, 2nd Edition, ACTEX, 1996, ISBN 1566982189
3. S. Broverman, *Mathematics of Investment and Credit Solutions Manual* 2nd edition, ACTEX, 1996, ISBN 1566982197
4. M. Sherris, *Money and Capital Markets: Pricing, Yields and Analysis*, 2nd Edition, Allen & Unwin, 1996, ISBN 1864481595