SUBJECT 103: STOCHASTIC MODELLING

Aim
The aim of the Stochastic Modelling course is to provide grounding in stochastic processes and their application to the models used for actuarial work.

Objectives
On completion of the course the trainee actuary will be able to:

(i) Describe the principles of actuarial modelling.
1. Describe why and how models are used.
2. Explain the benefits and limitations of modelling.
3. Explain the relative suitabilities of deterministic and stochastic models.
4. Describe, in general terms, how to decide whether a model is suitable for any particular application.
5. Explain the difference between the short-run and long-run properties of a model, and how this may be relevant in deciding whether a model is suitable for any particular application.
6. Describe, in general terms, how to analyse the potential output from a model, and explain why this is relevant to the choice of model.

7. Describe the process of sensitivity testing of assumptions and explain why this forms an important part of the modelling process.
8. Explain the factors that must be considered when communicating the results following the application of a model.

(ii) Describe the general principles of stochastic processes, and their classification into different types.
1. Define in general terms a stochastic process.
2. Classify a stochastic process according to whether it:
- operates in continuous or discrete time
- has a continuous or a discrete state space
- is a mixed type

and give examples of each type of process.
3. Describe possible applications of mixed processes.
4. Explain what is meant by the Markov property in the context of a stochastic process.
5. Describe the concepts of Martingales, filtration and stopping times as applied to stochastic processes.

(iii) Define and apply a Markov chain.
1. State the essential features of a Markov chain model.
2. State the recurrence relations that represent a Markov chain.
3. Calculate the stationary distribution for a Markov chain in simple cases.
4. Describe a system of frequency based experience rating in terms of a Markov chain and describe other simple applications.
5. Demonstrate how Markov chains can be used as a tool for modelling and how they can be simulated.

(iv) Define and apply a Markov process.
1. State the essential features of a Markov process model.

2. Define a Poisson process, derive the distribution of the number of events in a given time interval, derive the distribution of inter-event times, and apply these results.
3. Derive the Kolmogorov equations for a Markov process with time independent and time/age dependent transition intensities.
4. Solve the Kolmogorov equations in simple cases.
5. State difference equations that may be used to solve the Kolmogorov equations numerically.
6. State a simple numerical algorithm that could be used to solve the equations in more complex cases.
7. Describe simple survival models, sickness models and marriage models in terms of Markov processes and describe other simple applications.
8. State the Kolmogorov equations for a model where the transition intensities depend not only on age/time, but also on the duration of stay in one or more states.
9. Describe sickness and marriage models in terms of duration dependent Markov processes and describe other simple applications.
10. Demonstrate how Markov jump processes can be used as a tool for modelling and how they can be simulated.

(v) Define and apply the main concepts underlying the analysis of time series models.
1. Explain the concept and general properties of stationary, I(0), and integrated, I(1), univariate time series.
2. Explain the concept of a stationary random series.
3. Explain the concept of a filter applied to a stationary random series.
4. Know the notation for backwards shift operator, backwards difference operator, and the concept of roots of the characteristic equation of time series.
5. Explain the concepts and basic properties of autoregressive (AR), moving average, (MA), autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) time series.
6. Explain the concept and properties of discrete random walks and random walks with normally distributed increments, both with and without drift.
7. Explain the basic concept of a time series in the frequency domain.
8. Explain the basic concept of a multivariate autoregressive model.
9. Explain the concept of co-integrated time series.
10. Show that certain univariate time series models have the Markov property and describe how to rearrange a univariate time series model as a multivariate Markov model.
11. Outline the processes of identification, estimation and diagnosis of a time series, the criteria for choosing between models and the diagnostic tests that might be applied to the residuals of a time series after estimation.
12. Explain the basic concept of a transfer function model.
13. Describe briefly other non-stationary, non-linear time series models.
14. Describe simple applications of a time series model, including random walk, autoregressive and co-integrated models as applied to investment variables.

(vi) Define and apply the main concepts of Gauss-Wiener processes, and be aware of other Lévy processes.
1. Explain the definition and basic properties of univariate Brownian motion or Wiener process.
2. Show that the discrete sampling of a Wiener process gives a normally distributed random walk, and how it can also be seen as the limit of such a random walk.
3. Explain the concept of more general Gauss-Wiener processes.

4. Explain the definition and basic properties of a univariate Ornstein-Uhlenbeck process and show the relationship between this process and an AR(1) time series.
5. Explain the concept of a function of a Wiener process and show how the derivative and integral may be found.
6. Explain the concept of multivariate, possibly correlated, Wiener processes.
7. Explain the basic concept of a lognormal Wiener process.
8. Explain the basic concepts of stable distributions as applied to stochastic processes.
9. Describe simple applications of Lévy processes, including the lognormal Wiener process and the Ornstein-Uhlenbeck process for investment variables.
10. Demonstrate how Brownian motions and Itô processes can be used as a tool for modelling and how they can be simulated.

(vii) Explain the concepts of "Monte Carlo" simulation of a stochastic process using a series of pseudo-random numbers.
1. Describe how apparently pseudo-random integers can be generated using a computer.
2. Describe how pseudo-random drawings from specified distributions can be generated.
3. Explain how a series of sets of correlated normal random variates can be generated.
4. Explain the disadvantages of using truly random, as opposed to pseudo-random, numbers.
5. Explain the circumstances in which the same set of random numbers would be used for two sets of simulations and the circumstances in which different sets would be used.
6. Discuss how to decide how many simulations to carry out for any particular purpose.

The Main Reading: UK Institute of Actuaries Core Reading for subject 103 Stochastic Modelling

Additional Reading:

1. Brzezniak, Zdzislaw and Zastawniak, Tomasz, *Basic stochastic processes: A course through exercises.* Springer, 1998, ISBN 3540761756
2. Grimmett, Geoffrey and Stirzaker, David, *Probability and random processes.* 3rd ed. Oxford University Press, 2001, ISBN 0198572220
3. Grimmett, Geoffrey and Stirzaker, David, *One thousand exercises in Probability.* 2nd ed. Oxford University Press 2001, ISBN 0198572212
4. Norris, *Markov Chains.* Cambridge Uni Press 1997 ISBN 0521633966
   James D. Hamilton, Time Series Analysis. Princeton Univ Pr; 1994 ISBN: 0691042896