SUBJECT 105: ACTUARIAL MATHEMATICS 1 (LIFE INSURANCE)

Aim

The aim of the Actuarial Mathematics 1 course is to provide grounding in the mathematical techniques which are of particular relevance to actuarial work in life insurance, health and care and pensions.

Objectives

On completion of the course the trainee actuary will be able to:

(i) Define and use straightforward functions involving two lives.

1. Define the joint life functions l_{xy} and μ_{xy}.

2. Derive formulae for and evaluate the probabilities of death or survival of either or both of two lives.

3. Derive formulae for and evaluate the joint life and last survivor functions of $A_{xy}, A_{\overline{xy}}, a_{xy}, a_{\overline{xy}}, a_{x|y}$ and A^1_{xy} and their continuous equivalents, including consideration of them as expected values of functions of random variables.

4. Derive formulae for and evaluate the variances of the present values corresponding to the functions in 3.

5. Extend the techniques in 3 to deal with:
   - continuous functions
   - annuities payable more frequently than annually, by extending the techniques of Subject 104
   - functions dependent upon term as well as age

(ii) Define, estimate and use straightforward functions involving selection.

1. Describe how the techniques in Objective (vi) of Subject 104 can be extended to situations where the transition intensities depend on both age and duration.

2. Describe the select life table functions $l_{[x]+r}$ and $d_{[x]+r}$ and the ultimate life table functions l_x and d_x.

3. Define the following probabilities: ${}_{n|m}q_{[x]+r}$, ${}_{n|}q_{[x]+r}$.

4. Express the following life table probabilities in terms of the functions in 2: ${}_{n}p_{[x]+r}$, ${}_{n}q_{[x]+r}$, ${}_{n|m}q_{[x]+r}$.

5. Calculate net premiums and net premium policy values using select mortality.

(iii) Describe the main variable benefit, disability and long-term care contract types and calculate net premiums and reserves for them.

1. Describe the following contracts
   - with profit policies
   - index-linked annuities
   - unit-linked contracts
   - salary-related policies (including policies to provide pensions)
   - disability contracts
   - long-term care contracts

2. State the main objectives of these contracts.

3. Develop the net future loss random variable for the contracts in 1, assuming deterministic interest, inflation, bonus, and salary escalation assumptions.

4. Calculate the net premium for the contracts in 1 using commutation functions or by a simple life table approach.

5. Calculate the net premium prospective reserve for the contracts listed.

(iv) Describe the types of future expenses and bonus required for pricing and reserving and the influence of inflation on these.

1. List the types of expenses incurred in writing a life insurance contract.
2. Describe the influence of inflation on the expenses listed in 1.
3. Describe the types of bonus that may be given to a with profits contract.

(v) Describe the calculation of gross premiums and reserves using the equation of value for fixed benefit and variable benefit contracts.

1. Define the gross future loss random variable for standard contract types.
2. Calculate the gross premium using the future loss random variable and the equivalence principle, for premiums payable annually, more frequently than annually and continuously, and for benefits payable annually or at the end of the year of death, or continuously or immediately on death.
3. Calculate the gross premium using simple criteria other than the equivalence principle.
4. Calculate the gross premium prospective reserve using the future loss random variable.
5. Define and calculate the gross premium retrospective reserve.
6. State the conditions under which, in general, the prospective reserve is equal to the retrospective reserve allowing for expenses.
7. Prove that, under the appropriate conditions, the prospective reserve is equal to the retrospective reserve, with or without allowance for expenses, for all standard fixed benefit and variable benefit contracts.
8. Derive a recursive relation between successive annual reserves for an annual premium contract, with allowance for expenses, for standard fixed benefit contracts.
9. Derive the gross premium reserve in terms of the net premium reserve with Zillmer adjustment, assuming that the gross and net premiums and reserves are calculated using the same interest and mortality assumptions.
10. Explain why the Zillmerised reserve may be used in place of the net premium reserve even if the conditions in 9 do not hold.

(vi) Describe the technique of discounted emerging costs, for use in pricing, reserving, and assessing profitability, for all contract types and for pensions.

1. Evaluate expected cash flows for whole life, endowment and term assurances, annuities, unit-linked contracts and disability and long-term care contracts.
2. Profit test simple annual premium contracts of the types listed in 1 and determine the profit vector, the profit signature, the net present value, and the profit margin.
3. Show how the profit test may be used to price a product.
4. Explain why it is necessary to zeroise negative cash flows.
5. Show how the profit test may be used to determine reserves.
6. Explain the purpose of actuarial funding for unit linked contracts and carry out profit tests of simple contracts using actuarial funding.
7. Explain why pricing and reserving bases may be different.
8. Demonstrate how the profit test will be affected by the choice of pricing and reserving bases.
9. Describe the construction and use of a multiple decrement service table for pension calculations, including the relationships with associated single decrement tables.
10. Use the service table and salary scale to evaluate expected cash flows for individual salary-related pensions benefits and contributions and describe how commutation functions may be developed as a computational tool.

(vii) Describe the technique of asset shares in the context of life insurance contracts and the relationship of the asset share to the retrospective reserve.

1. Explain how an asset share may be built up using a recursive formula.
2. Explain the relationship between the asset share and the retrospective reserve.
3. Explain how the asset share may be used in the determination of bonus distribution.

(viii) Calculate the benefits on the early termination of a contract, including transfer, and the premium or benefits after a change in the terms of a contract.

1. Calculate surrender values for conventional insurance contracts using policy values or the future loss random variable.
2. Calculate paid-up sums assured for conventional insurance contracts using policy values or the future loss random variable.
3. Use the policy values to evaluate the financial effect of alterations to policies.
4. Describe the benefit options available to an individual leaving a pension scheme.
5. Calculate the transfer cash equivalent for an individual leaving a pension scheme.

(ix) Describe the calculation of the cost of guarantees and options under life insurance contracts.

1. Describe the main types of investment guarantee and mortality option that may be given under life insurance contracts.
2. Explain how option-pricing techniques or stochastic simulation can be used to calculate the cost of an investment guarantee.
3. Describe how the cost of a simple mortality option can be calculated using the conventional method or the North American method.

(x) Describe the principal forms of heterogeneity within a population and the ways in which selection can occur.

1. State the principal factors which contribute to the variation in mortality and morbidity by region and according to the social and economic environment, specifically:
   - occupation
   - nutrition
   - housing
   - climate/geography
   - education
2. Define and give examples of the main forms of selection:
   - temporary initial selection
   - class selection
   - time selection
   - spurious selection
   - adverse selection
3. Explain how selection can be expected to occur amongst lives taking out each of the main types of life insurance contracts, or amongst members of large pension schemes.
4. Explain why it is necessary to have different mortality tables for different classes of lives.
5. Explain how decrements can have a selective effect.
6. Explain the theoretical basis of the use of risk classification in life insurance.
7. Explain the concept of a single figure index and its advantages and disadvantages for summarising and comparing actual experience.
8. Define the terms crude index, direct standardisation and indirect standardisation, standardised mortality/morbidity rate, Standardised Mortality/Morbidity Ratio (SMR) and illustrate their use.

(xi) Describe the process of population projection and its main determinants.

1. Describe the use of simple mathematical models of population projection and their shortcomings.
2. Describe the component method of population projection.
3. List the socio-economic factors that affect fertility and describe how fertility rates might be projected.

xii) Derive computational tools for use in determining the value of the benefits under a disability insurance contract.

1. Describe how a disability benefit may be valued numerically using a multiple state model.
2. Describe how a disability benefit may be valued using rates of claim inception and claim termination.
3. Define and use "Manchester-Unity" type disability functions.

The Main Reading: UK Institute of Actuaries Core Reading for subject 105 Actuarial Mathematics 1

Additional Reading:

1. Bowers, Newton L et al., *Actuarial mathematics* – 2nd ed. Society of Actuaries, 1997 ISBN
2. Neill, Alistair, *Life contingencies.* Heinemann, 1977, 452 pages. ISBN 0434914401.
3. Gerber, H. U., *Life insurance mathematics* – 3rd ed. Springer. Swiss Association of Actuaries, 1997 ISBN 354062242X.
4. Benjamin, Bernard and Pollard, John H., *The analysis of mortality and other actuarial statistics.* 3rd ed. Institute of Actuaries and Faculty of Actuaries, 1993 ASIN: 0750608501
5. Haberman, S. and Pitacco, E., *Actuarial models for disability insurance* - Chapman & Hall, 1999 ISBN: 0849303893
6. Booth, P. M et al., *Modern actuarial theory and practice* - Chapman & Hall, 1999 ISBN: 0849303885