SUBJECT 106: ACTUARIAL MATHEMATICS 2 (NON-LIFE INSURANCE)

Aim
The aim of the Actuarial Mathematics 2 course is to provide grounding in the mathematical techniques, which are of particular relevance to actuarial work in non-life insurance.

Objectives
On completion of the course the trainee actuary will be able to:

(i) Explain the concepts of decision theory and apply them.
1. Determine optimum strategies under the theory of games.
2. Explain what is meant by a decision function and a risk function.
3. Apply decision criteria to determine which decision functions are best with respect to a specified criterion. In particular consider the minmax criterion and the Bayes criterion.

(ii) Explain the fundamental concepts of Bayesian statistics and use these concepts to calculate Bayesian estimators.
1. Use Bayes Theorem to calculate simple conditional probabilities.
2. Explain what is meant by a prior distribution, a posterior distribution and a conjugate prior distribution.
3. Derive the posterior distribution for a parameter in simple cases.
4. Explain what is meant by a loss function.
5. Use simple loss functions to derive Bayesian estimates of parameters.

(iii) Calculate probabilities and moments of loss distributions both with and without simple reinsurance arrangements.
1. Describe the properties of the statistical distributions, which are suitable for modelling individual and aggregate losses.
2. Derive moments and moment generating functions (where defined) of loss distributions including the gamma, exponential, Pareto, generalised Pareto, normal, lognormal, Weibull and Burr distributions.
3. Apply the principles of statistical inference to select suitable loss distributions for sets of claims.
4. Explain the concepts of excesses (deductibles), and retention limits.
5. Describe the operation of simple forms of proportional and excess of loss reinsurance.
6. Derive the distribution and corresponding moments of the claim amounts paid by the insurer and the reinsurer in the presence of excesses (deductibles) and reinsurance.

(iv) Construct risk models appropriate to short term insurance contracts and calculate the moment generating function and the moments for the risk models both with and without simple reinsurance arrangements.
0. Define the basic short-term contracts.
1. Construct models appropriate for short term insurance contracts in terms of the numbers of claims and the amounts of individual claims.
2. Describe the major simplifying assumptions underlying the models in 1.
3. Derive the moment generating function of the sum of N independent random variables; in particular when N has a binomial, Poisson, geometric or negative binomial distribution.
4. Define a compound Poisson distribution and show that the sum of independent random variables each having a compound Poisson distribution also has a compound Poisson distribution.

5. Derive the mean, variance and coefficient of skewness for compound binomial, compound Poisson and compound negative binomial random variables.
6. Derive formulae for the moment generating functions and moments of aggregate claims over a given time period for the models in 1. in terms of the corresponding functions for the distributions of claim numbers and claim amounts, stating the mathematical assumptions underlying these formulae.
7. Repeat 5 for both the insurer and the reinsurer after the operation of simple forms of proportional and excess of loss reinsurance.

(v) Calculate and approximate the aggregate claim distribution for short term insurance contracts.
1. Derive a recursion formula for calculating the aggregate claim distribution in cases where claim amounts are distributed on the positive integers and claim numbers have a binomial, a Poisson or a negative binomial distribution.
2. Approximate the aggregate claim distribution by a normal distribution fitted by moments.
3. Approximate the aggregate claim distribution by a translated gamma distribution fitted by moments.

(vi) Explain the concept of ruin for a risk model. Calculate the adjustment coefficient and state Lundberg's inequality. Describe the effect on the probability of ruin of changing parameter values and of simple reinsurance arrangements.
1. Explain what is meant by the aggregate claim process and the cash-flow process for a risk.
2. Define the probability of ruin in infinite/finite and continuous/discrete time and state and explain relationships between the different probabilities of ruin.
3. Define a Poisson process, derive the distribution of the number of events in a given time interval, derive the distribution of inter-event times, and apply these results.
4. Define a compound Poisson process and derive the moments and moment generating function for such a process.
5. Define the adjustment coefficient for a compound Poisson process and for discrete time processes, which are not compound Poisson, calculate it in simple cases and derive an approximation.
6. State Lundberg's inequality and explain the significance of the adjustment coefficient.
7. Describe the effect on the probability of ruin, in both finite and infinite time, of changing parameter values.
8. Analyse the effect on the adjustment coefficient and hence on the probability of ruin of simple reinsurance arrangements.

(vii) Describe and apply the fundamental concepts of credibility theory.
1. Explain what is meant by the credibility premium formula and describe the role played by the credibility factor.
2. Explain the Bayesian approach to credibility theory and use it to derive credibility premiums in simple cases.
3. Explain the empirical Bayes approach to credibility theory, in particular its similarities with and its differences from the Bayesian approach.
4. Use the empirical Bayes approach to derive credibility premium formulae for the two standard elementary models of empirical Bayes credibility theory, one incorporating and one not incorporating risk volumes.
5. State the assumptions underlying the two models in 4.
6. Calculate credibility premiums for the two models in 4.

(viii) Describe the fundamental concepts of rating general insurance business and apply them to simple experience rating systems.

0. Describe the basic methodology used in rating general insurance business.
1. Explain the workings of a simple experience rating system based on claim frequency and calculate the stationary distribution for the number of policyholders at each level of discount.
2. Calculate the probability, using simple criteria, that the policyholder will make a claim.
3. Repeat 1, applying the results from 2.

(ix) Describe and apply techniques for analysing a delay (or run-off) triangle and projecting the ultimate position.
1. Define a development factor and show how a set of assumed development factors can be used to project the future development of a delay triangle.
2. Describe and apply the basic chain ladder method for completing the delay triangle.
3. Show how the basic chain ladder method can be adjusted to make explicit allowance for inflation.
4. Discuss alternative ways for deriving development factors, which may be appropriate for completing the delay triangle.
5. Describe and apply the average cost per claim method for estimating outstanding claim amounts.
6. Describe and apply the Bornhuetter-Ferguson method for estimating outstanding claim amounts.
7. Discuss the assumptions underlying the application of the methods in 1 to 6 above.

(x) Explain the fundamental concepts of a generalised linear model (GLM), and describe how a GLM may be applied.
1. Define an exponential family of distributions. Show that the following distributions may be written in this form: binomial, Poisson, exponential, gamma, normal.
2. State the mean and variance for an exponential family, and define the variance function and the scale parameter. Derive these quantities for the distributions in 1.
3. Explain what is meant by the link function and the canonical link function, referring to the distributions in 1.
4. Explain what is meant by a variable, a factor taking categorical values and an interaction term. Define the linear predictor, illustrating its form for simple models, including polynomial models and models involving factors.
5. Define the deviance and scaled deviance and state how the parameters of a GLM may be estimated. Describe how a suitable model may be chosen by using an analysis of deviance and by examining the significance of the parameters.
6. Define the Pearson and deviance residuals and describe how they may be used.

The Main Reading:UK Institute of Actuaries Core Reading for subject 106 Actuarial Mathematics2

Additional Reading:

1. Bowers, Newton L et al., *Actuarial mathematics* – 2nd ed. Society of Actuaries, 1997 ISBN
2. R. Kaas Marc Goovaerts, Jan Dhaene, Michel Denuit, *Modern actuarial risk theory.* Kluwer Academic Publishers, 2001 ISBN 0792376366
3. Klugman, S. A et al., *Loss models: from data to decisions* - John Wiley, 1998 ISBN**:** 0471238848
4. *Foundations of casualty actuarial science* - Casualty Actuarial Society – 4th ed. - CAS,
5. Straub, E., *Non-Life Insurance Mathematics.* Springer 1988 ISBN: 0387187871
6. Booth, P. M et al., *Modern actuarial theory and practice* - Chapman & Hall, 1999 ISBN: 0849303885
7. Daykin, C. D; Pentikainen, T.; Pesonen, M., *Practical risk theory for actuaries* – Chapman & Hall, 1994 ISBN: 0412428504
8. Jan Grandell, *Aspects of Risk Theory.* Springer 1991 – ISBN: 0387973680

9. Taylor, G., *Loss Reserving - An Actuarial Perspective.* Kluwer Academic Publishers 1999 ISBN 0792385020
10. Jan Grandell, *Aspects of Risk Theory* - Springer Verlag, 1991 ISBN: 0387973680