SUBJECT 109: FINANCIAL ECONOMICS

Aim
The aim of the Financial Economics course is to develop the necessary skills to construct and apply asset liability models and to value financial derivatives. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

Objectives
On completion of the course the trainee actuary will be able to:

ASSET PRICING AND PORTFOLIO SELECTION MODELS

(i) Describe and discuss the application of utility theory to financial problems.
1. Explain the meaning of the term "utility function".
2. Explain the axioms underlying utility theory and the expected utility theorem.
3. Explain how the following economic characteristics of investors can be expressed mathematically in a utility function:
   - non-satiation
   - risk aversion, risk neutrality and risk seeking
   - declining or increasing absolute and relative risk aversion
4. Discuss the economic properties of commonly used utility functions.
5. Discuss how a utility function may depend on current wealth and discuss state dependent utility functions.

(ii) Describe and discuss the application of stochastic dominance to portfolio selection problems.
1. State the conditions for absolute dominance and for first, second and third order stochastic dominance.
2. Discuss the relationships between dominance concepts and utility theory.

(iii) Discuss the advantages and disadvantages of different measures of investment risk.
1. Define the following measures of investment risk:
   - variance of return
   - downside semi-variance of return
   - shortfall probabilities
2. Describe how the risk measures listed in (iii) 1 above are related to the form of an investor's utility function.

(iv) Describe and discuss the assumptions of mean-variance portfolio theory and its principal results.
1. Describe and discuss the assumptions of mean-variance portfolio theory.
2. Discuss the conditions under which application of mean-variance portfolio theory leads to the selection of an optimum portfolio.
3. Calculate the expected return and risk of a portfolio of two risky assets, given the expected return, variance and covariance of returns of the individual assets, using mean-variance portfolio theory.
4. Explain how the expected return and risk of a portfolio of many risky assets is calculated using mean-variance portfolio theory.

5. Explain what is meant by: opportunity set, efficient frontier, indifference curves and the optimum portfolio, in the context of mean-variance portfolio theory.
6. Derive expressions for the efficient frontier and explain how to find an optimum portfolio in the absence of constraints.
7. Explain the use of portfolio theory when there are constraints on the proportion invested in each security.
8. Explain how the efficient frontier and the optimum portfolio changes with the introduction of risk-free lending and borrowing.
9. Show how the "beta factor" of a security can be calculated relative to an efficient portfolio.

(v) Describe and discuss the properties of single and multifactor models of asset returns.
1. Describe the three types of multifactor models of asset returns:
- macroeconomic models
- fundamental factor models
- statistical factor models

2. Discuss the single index model of asset returns.
3. Discuss the concepts of diversifiable & non-diversifiable risk.
4. Discuss the construction of the different types of multifactor models.

(vi) Describe how liabilities can be incorporated into portfolio selection models and describe the process of asset liability modelling.
1. Discuss how liabilities can be incorporated into portfolio selection models.
2. Outline the process of asset liability modelling and discuss the main factors that must be considered at each stage:
- specification of the objectives and time horizon
- choice of the objective function and the variables over which it is to be optimised
- running of simulations
- mapping of the efficient opportunity set and distribution of possible outcomes
- presentation of the results and choice of the optimal strategy

3. Demonstrate an understanding of the concept of a state-price (Arrow-Debreu) security and of state price deflators.

(vii) Describe equilibrium models, such as the Capital Asset Pricing Model, discussing the principal results and assumptions and limitations of such models.
1. Describe the assumptions of the CAPM.
2. Discuss the principal results of the CAPM.
3. Discuss the limitations of the basic CAPM and some of the attempts that have been made to alter the theory to overcome these limitations.
4. Discuss the assumptions and principal results of the Arbitrage Pricing Theory model.

STOCHASTIC ASSET MODELS

(viii) Discuss the various forms of the Efficient Markets Hypothesis and discuss the evidence for and against the hypothesis.
1. Discuss the three forms of the Efficient Markets Hypothesis and their consequences for investment management.
2. Describe briefly the evidence for or against each form of the Efficient Markets Hypothesis.

(ix) Demonstrate a knowledge and understanding of stochastic models of the behaviour of security prices.

1. Discuss the continuous time log-normal model of security prices and the empirical evidence for or against the model.
2. Discuss auto-regressive models of security prices and other economic variables, such as the Wilkie model, and describe the economic justification for such models.
3. Discuss the main alternatives to the models covered in (ix) 1 and (ix) 2 above and describe their strengths and weaknesses.

(x) Describe the problems involved in estimating parameters for asset pricing models.
1. Discuss the main problems involved in estimating parameters for asset pricing models:
- data availability
- data errors
- outliers
- lack of stationarity of underlying time series
- the role of judgement

(xi) Demonstrate a knowledge and understanding of models of the term structure of interest rates.
1. Explain the differences between no arbitrage models of the term structure of interest rates and the general equilibrium approach and the conditions under which the two approaches are equivalent.
2. Explain the basic (one-factor) arbitrage pricing theory model (Vasiček) together with the assumptions on which it is based.
3. Explain the basic (one-factor) general equilibrium theory model (Cox-Ingersoll-Ross) together with any assumptions on which it is based.
4. Outline the deficiencies of the two basic models together with possible modifications.

VALUATION OF DERIVATIVE SECURITIES

(xii) Demonstrate a knowledge and understanding of the properties of option prices, factors affecting option prices, upper and lower bounds for option prices.
1. Outline the factors that affect option prices.
2. Develop upper and lower bounds for European and American call and put options.
3. State what is meant by put-call parity.
(xiii) Demonstrate a knowledge and understanding of the Black-Scholes analysis and arbitrage free pricing.
1. Show an understanding of the derivation of the Black-Scholes option pricing formula
1.1 Explain what is meant by a complete market, and by risk-neutral valuation and equivalent martingale measure.
1.2 Derive the Black-Scholes differential equation in the Garman-Kohlhagen form.
1.3 Discuss the validity of the assumptions underlying the Black-Scholes differential equation.
2. Demonstrate an understanding of the Black-Scholes pricing formula in the Garman-Kohlhagen form for valuing a European call option on a stock index paying dividends continuously and show how to apply it.
3. Describe the way in which the formula can be adjusted (including the nature of any approximations) to value:
- European put options
- American call options
- options on shares that pay dividends discretely
4. Demonstrate an awareness of the problems in pricing American put options.
5. Describe how the Black-Scholes pricing formula can be used to determine implied volatility.

6. Demonstrate an awareness of the commonly used terminology for the first, and where appropriate second, partial derivatives of an option price with respect to:
- the price of the underlying security
- the volatility of the price of the underlying security
- the risk free interest rate
- time to expiry
- running yield

7. Describe the characteristics of the partial derivatives listed above and derive expressions for them from the Black-Scholes formula in the Garman-Kohlhagen form.
8. Demonstrate an understanding of the concept of delta-hedging and show how to apply it.

(xiv) Demonstrate knowledge and understanding of numerical procedures used in derivative pricing and valuation.
1. Describe the use of binomial trees and lattices in valuing options and solve simple examples.
2. Show how approximate values of the partial derivatives listed in (xiii) 6 can be derived from a binomial lattice.

The Main Reading: UK Institute of Actuaries Core Reading for subject 109 Financial Economics

Additional Reading:

1. Panjer, H. et.al, *Financial Economics*. SOA, 1998.
2. Hull John C., *Options, futures and other derivatives*. 4th ed. Prentice Hall, 2000 ISBN 013 015822 4
3. Chi-Fu Huang, Robert H. Litzenberger, *Foundations for Financial Economics*. McGraw-Hill/Appleton & Lange; 1998 ISBN 0444013105
4. Pliska, S., *Introduction to Mathematical Finance: Discrete Time Models*. Blackwell Publishers; (June 1997) ISBN: 1557869456
5. Alison Etheridge, *A Course in Financial Calculus.* Cambridge Uni Press 2002 ISBN: 0521890772
6. Cochrane, J., *Asset Pricing.* Princeton University Press, 2001 ISBN: 0691074984
7. Salih Neftci *An Introduction to the Mathematics of Financial Derivatives.* 2nd ed. Academic Press, 2000 ISBN 0125153929
8. Yuh-Dauh Lyuu *Financial Engineering and Computation.* Cambridge Uni Press, 2002 ISBN 052178171X
9. Weron, A. and Weron, R. *Inzyneria finansova.* WNT Warszawa, 1999 ISBN 8320424712